

# Mathematical Modeling of Facility Location for Bank Earnings Maximization when Users Rank Facilities by Shorter Travel and Waiting Times

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## ABSTRACT

In this study, it has been tried to determine the optimal location criteria of bank with Case Study Post Bank in the region's seven of Tehran, Iran, with two main goals first gaining the largest market share and second, minimizing the distance between the customers' access to bank. The optimal location criteria of bank was developed with four criteria include travel time, waiting time, service quality, reliability and warranty and advertising, which set by a questionnaire with AHP method of client. The population of the study has been included 60 randomly clients divided into five class from seven regional. The result has been showed that both travel time and waiting time are the highest rated. Therefore, based on these two criteria, the employer has been subjected to the maximum benefit of the branches. The research's model has been solved using MATLAB software. The study area was divided into 81 nodes and the necessary data were obtained from the branches and seven regional council. Finally, the results has been showed that the model presented in this study are able to predict the location of optimum places.

## KEYWORD

Mathematical Modeling, Shorter Travel, Waiting Times, MATLAB software.

## INTRODUCTION

Providing the appropriate service, high quality of service, quick performance, and competitive prices are important for an enterprise, but all they are affected by the location factor because in the case of improper selection of commercial unit location, all factors will be affected and their positive effects will be reduced (Loken (2007); Zhao et al. (2004)). Thus, proper location creates a strategic advantage compared to the competitors. Therefore, the establishment of enterprise in appropriate locations and in

the best possible status will develop the target business in addition to preventing the capital loss (according to the constant constraint of capital and time); moreover, it will improve the commodity/service flow to the consumers and put the enterprise businesses in an ideal status (Li et al. 2013). In banking industry like other industries, there are other competitive factors in addition to the competition for the cost of service providing; the way of providing the service for the customer is the most important competitive factor (Jesse et al. 2013; Berjisian, 2006; Loken, 2007). In this regard, the number and location of bank branches is so important because they determine the extent and ease of access to banking services cover more places and attract more potential customers in the branches. Moreover, the costs, related to the need for movement to reach the nearest branch, are decreased (Hopmans, 1986; Jesse et al. 2013). Thus, the number of customers' referring to the bank will be increased and they use more services. In this paper, the district 7 of Tehran city is studied based on the above definition with the suggestion by Post bank officials. The target district is classified into multiple regions as follows, and in this case, the district is converted to a discrete space. Then the locations of competitor' banks are identified and investigated and the demand rate is estimated, and then the optimal locations are determined for establishing the bank branches by providing the appropriate model based on the mathematical programming and problem solving by the help of ultra-innovative methods (Jesse et al. 2013; Tang et al. 2013; Loken, 2007). Finally, it can argue that the main objective of study is to determine the optimal location criteria of bank as well as determining the optimal location of bank. In this regard, the model of study seeks to achieve two main objectives of the largest market share and minimizing the distance of customer's access to bank .

## 1-1- REVIEW OF LITERATURE

In the field of bank branches location, first Cornuejols et al. (1977) studied the issue of bank branches location in order to maximize the use of time for transferring the funds between banks. By providing an interactive distance model

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between the branches of banks, Knut and Sven (2013) expressed the attractiveness of bank branches and the retrograde distance function as the effective factors in location. Mousavi (2001) considered in his study the optimal points such as the crossroads, squares, etc. as the evaluated potential sites and the best places for establishing the branches and then chose the optimal points for establishment of bank branches in district 16 of Tehran through the Geographic Information System. Tang et al. (2013) determined the optimal points for establishment of private bank branches in Tehran in his study through the geographic information systems. Goli et al. (2010) provided a framework for location of bank ATMs by using the spatial analysis and the hierarchical analysis method. Aldajani et al. (2009) introduced a framework for location of bank ATMs by providing the algorithm for solving the mathematical model. However, none of the studies has paid attention to the bank location. Therefore, the researcher in this study is seeking to estimate the optimal model and solve the model for bank location with the case study of Post Bank of Iran due to such a scientific gap.

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## 2- PRESENTING THE MATHEMATIC MODEL OF THREE-PHASE MULTI-PRODUCT SUPPLY CHAIN NETWORK

The target case study in this research is to establish Post Bank branch in district 7 of Tehran. Figure (1) shows the location of this district. The GPS maps of the year 2011 were used for this district due to the lack of updated maps, which show the available branches of bank. Based on this map, the number of bank branches is 128 branches in this region regardless of the type of bank. Software MATLAB.V2011 was used for solving the mathematical model by the simulated annealing algorithm (SA) was applied for obtaining the optimal response. The presented model in this study considers the location issue of bank branches according to two viewpoints. First, the employer's viewpoint or the studied bank that aims to maximize the profit of target function and second, the customer's viewpoint who wants no wasted money and time for waiting in queue and traveling to the branch location. Therefore, the aim of target function is to maximize the profit from the bank activities with the customers' cash assets. In calculating the target function for estimating the rate of bank profit, the bank revenue sources have been taken into account. Customers' deposits are the sources of entering the bank assets and assigning the facilities as well as the commercial investment are the source of bank profit. Thus, the bank profit is obtained from the difference between the profit of assigning the facilities and the bank investment in addition to the bank interest paid to the customers' deposits and the current costs of branch. The values  $\beta_1, \beta_2, \beta_3$  are the interest of assigning the facilities, investment interest, and interest of customers' deposits, respectively.  $\varphi$  is the choice between the facilities and investment. Therefore, the target function can be written as Eq (1).

$$\max z = \sum_{j \in N \cup N'} \{(\beta_1 \varphi + \beta_2 (1 - \varphi)) s_i \lambda_j - f_j y_j - \beta_3 h_j \lambda_j\} \quad (1)$$

$s_i$  : Average cash assets rate in demand node  $i$ ;

$\lambda_j$  : Average customer entering rate to the node  $j$  of facilities

$f_j$  : Current cost of branch at the node  $j$ ;

$y_j$  : Decision variable which is equal to 1 if the facilities are located at the node  $j$ , otherwise it will be equal to 0.

$h_j$  : Average demand rate at each demand node  $j$ ;

In this model, two sets have been selected as follows:

$N$  : The set of candidate locations for establishment of bank branches where there is no rival banks there.

$N'$  : The set of candidate locations for establishment of bank branches where there is the rival banks there.

Thus, each node belongs to  $N \cup N'$  in this model can be as a candidate for establishing the bank and the decision variable  $y_j$  makes decision in this regard. We can show  $x_{ij}$ , which is the possibility to choose the customer of node  $i$  for going to the facility node  $j$ , and  $v_{ij}$ , which is the possibility to choose the bank branches at the node  $j$  in order to get the facilities by the customer of node  $i$ , as Eq (2):

$$x_{ij} = \frac{y_j e^{-\gamma C_{ij}}}{\sum_{k \in N} y_k e^{-\gamma C_{ik}} + \sum_{k \in N'} e^{-\gamma C_{ik}}}; v_{ij} = \frac{1}{b_j} x_{ij} \quad (2)$$

$b_j$  : Number of bank rival branches available at the node  $j$ ;

$C_{ij}$  is the agent of choosing the customer. This agent was prepared through the questionnaire AHP and based on the customers' feedback. The questionnaire was initially prepared according to the Appendix and then was provided for the statistical population. Based on the results, obtained from the AHP test, two factors of travel time and waiting time were identified with the highest rank; therefore, these two parameters were used for developing the model. To prevent the error, the reliability of questionnaire was examined and its accuracy was confirmed. Furthermore, the relevant questionnaire was randomly distributed among 60 individuals in five branches of banks in district 7 and the relevant data was collected. Finally, it was found that the highest proportion in choosing a bank depends on the travel time and waiting time in the queue. It should be noted that the information related to this stage of research has a high volume, which is larger than the limit of this dissertation, and the results have been only presented. Therefore,  $C_{ij}$ , which is the factor of choosing the customer, depends on the cost of traveling from the node  $i$  and the waiting cost at the node  $j$ . This dependence has been shown as a function as follows.  $\alpha$  is the customer's preference to choose between the cost of travelling or the waiting cost.  $\gamma$  is the indicator of customers' preferences which is defined as  $[\gamma = \pi / (\sigma \sqrt{6})]$  in which  $\sigma$  is the standard deviation of customer's preferences. If the amount of  $\gamma$  becomes greater, the customers' choice will become similar to each other and if it is smaller, the difference in choosing the customers will be increased.

$$C_{ij} = \alpha(\pi_j t_{ij}) + (1 - \alpha)(\theta_j w_j) \quad (3)$$

$\pi_j$ : Travel cost to the node  $j$  per time unit;

$t_{ij}$ : Customer's travel time at the node  $i$  to the candidate location of facilities at the node  $j$ ;

$\theta_j$ : Waiting cost at the node  $j$  per time unit;

$w_j$ : Average waiting time in the queue at the node  $j$ .

The average customer-entering rate to node  $j$  is equal to the average demand rate at each demand node  $i$  multiplied by the probability of choosing the customer at the same node, which can be formulated as follows:

$$\lambda_j = \sum_{i \in N \cup N'} h_i v_{ij} \quad (4)$$

For calculating the waiting time in the branch, we should calculate the queue length. Assuming the minimum queue length  $m$  and maximum queue length  $k$ , the equation of queue length can be written as follows.

$$L_j = \sum_{n=m}^k (n - m) p_{nj} \quad (5)$$

$$p_{nj} = \frac{\rho_j^n}{n!} p_{oj}, n \leq m \quad (6)$$

$$p_{nj} = \frac{\rho_j^n}{m! m^{n-m}} p_{oj}, m \leq n \leq k \quad (7)$$

$$p_{nj} = 0, n > k$$

$$p_{oj} = \left[ 1 + \sum_{n=1}^m \frac{\rho_j^n}{n!} + \frac{\rho_j^m}{m!} \sum_{n=m+1}^k \left( \frac{\rho_j}{m} \right)^{n-m} \right]^{-1} \quad (9)$$

$L_j$ : Average queue length at the node  $j$ ;

$p_{oj}$ : Probability of customer absence at the node  $j$  of facilities;

$p_{nj}$ : Probability of customer presence at the node  $j$  of facilities.

According to the mentioned cases, the final model of research can be written as follows.

$\bar{\lambda}_j$ : Effective rate of customers' entering to the node  $j$  of facilities

$\mu_j$ : Average service rate at the node  $j$ .

Finally, the research model is being presented as follow:

$$\max z = \sum_{j \in N \cup N'} (\beta_1 \varphi + \beta_2 (1 - \varphi)) s_i \lambda_j - \sum_{j \in N \cup N'} f_j y_j - \beta_3 h_i \lambda_j$$

s.t.

$$x_{ij} = \frac{y_j e^{-\gamma C_{ij}}}{\sum_{k \in N} y_k e^{-\gamma C_{ik}} + \sum_{k \in N'} e^{-\gamma C_{ik}}}; v_{ij} = \frac{1}{b_j} x_{ij} \quad (M-1)$$

$$C_{ij} = \alpha(\pi_j t_{ij}) + (1 - \alpha)(\theta_j w_j) \quad (M-2)$$

$$\lambda_j = \sum_{i \in N \cup N'} h_i v_{ij} \quad (M-3)$$

$$L_j = \sum_{n=m}^k (n - m) p_{nj} \quad (M-4)$$

$$p_{nj} = \frac{\rho_j^n}{n!} p_{oj}, n \leq m \quad (M-5)$$

$$p_{nj} = \frac{\rho_j^n}{m! m^{n-m}} p_{oj}, m \leq n \leq k \quad (M-6)$$

$$p_{nj} = 0, n > k$$

$$p_{oj} = \left[ 1 + \sum_{n=1}^m \frac{\rho_j^n}{n!} + \frac{\rho_j^m}{m!} \sum_{n=m+1}^k \left( \frac{\rho_j}{m} \right)^{n-m} \right]^{-1} \quad (M-8)$$

$$w_j = \frac{L_j}{\bar{\lambda}_j} \quad (M-10)$$

$$\bar{\lambda}_j = \lambda_j (1 - p_{kj}) \quad (M-11)$$

$$\rho_j = \frac{\lambda_j}{\mu_j} \quad (M-12)$$

$$\sum_{j \in N \cup N'} y_j \leq p \quad (M-13)$$

$$\pi_j t_{ij} < \text{Target Cost} \quad (M-14)$$

$$x_{ij} \in [0, 1] \quad (M-15)$$

$$\sum_j x_{ij} = 1, \forall i \quad (M-16)$$

$$\lambda_j \geq 0 \quad (M-17)$$

$$y_j \in \{0, 1\} \quad (M-18)$$

$$i, j \in N \cup N' \quad (M-19)$$

$$0 \leq \alpha \leq 1$$

Equation (M-10) calculates the waiting time in the branch, and the Equation (M-11) has been also used in the calculations in order to reduce the calculation error and consider the customer absence. In general, the Equations (M-5 to M-12) are for calculating the waiting time in the branch. Equations (M-13 to M-21) are the limitations of model. Equation 13 indicates that the maximum number of branches should be equal to  $p$ .

## 2-CASE STUDY DATA FOR OPTIMAL LOCATION

According to the research model and studied region, the location of study was divided into 81 sub-regions equal to 700x700 m<sup>2</sup>. Obtained diagram is shown in the following (Figure (1)).

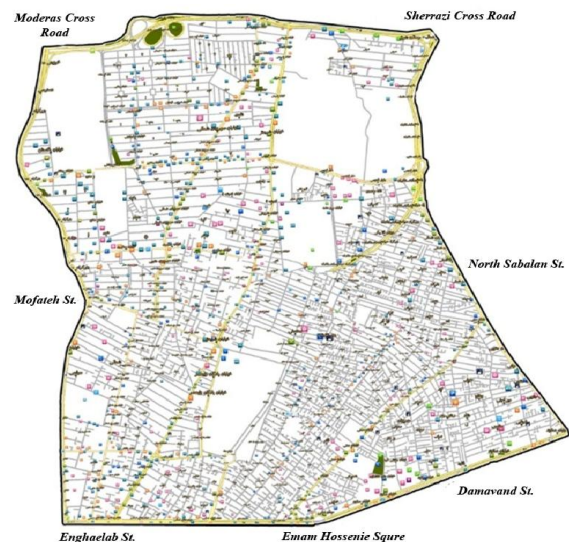


Fig.1. Diagram of reticulation and numbering the nodes of studied region

The data of target model was estimated for each of the available nodes. Variables of model were estimated as follows. Constant coefficients including  $\beta_1, \beta_2, \beta_3$ , which were the interest of assigning the facilities, investment interest, and customers' deposits interest, were estimated equal to 0.21, 0.32 and 0.17, respectively, from the average annual values of Post Bank on the official website of Bank. The amount of choice between the facilities and investment was estimated equal to  $\varphi = 0.3$ . Moreover, the index of customer preference for choosing the travelling cost or



waiting cost was assumed equal to  $\alpha = 0.6$ , the system capacity equal to  $k = 20$  and the number of servers was assumed equal to  $m = 6$ . Due to the lack of considering the difference among the brands of banks, the difference of preferences among the customers was considered equal to zero ( $\gamma=0$ ). Furthermore, the travelling cost was calculated according to the distance between the demand node and the facilities node with the assumed coefficient 500 Tomans per kilometer. Target cost was considered equal to 2000 Tomans; and the maximum travel cost (equal to four taxi courses) is considered if there is the limitation. Current cost of bank was considered as a source of bank costs and equal to six employees' annual salary which was 600 thousand Tomans on average. Other research variables including the number of rival banks, average demand rate, current costs, etc, were calculated for each separate node in the necessary conditions as follows. Numbers of rival banks were manually counted and reported through the GPS Map in each of 81 regions. The average demand rate was estimated equal to 25% of population in the target region according to the Central Bank statistics contained in annual report of Central Bank of Iran in 2010. Moreover, the population of each region was calculated by the data of municipality in district 7 and according to the Census data in 2010. After determining the input values for studied region, a computer program was written based on the research model in MATLAB programming environment; it is available in the Appendix

### 3- DETERMINING THE SIMULATED ANNEALING ALGORITHM PARAMETERS

In Simulated annealing method (SA), each point  $s$  is similar to a state of a physical system in searching space and the function  $E(s)$ , which should be minimized is similar to the internal energy of system in that state. This method aims to transfer the system from the arbitrary initial state to the state in which the system has the lowest energy. For solving an optimization problem, the SA algorithm first starts with an initial response and then moves in a loop towards neighboring responses. If the neighboring response is better than the current solution, the algorithm chooses it as the current response (move towards it), otherwise the algorithm accepts that solution as the current response with the probability equal to  $\exp\left(-\frac{\Delta E}{T}\right)$ . In this relation,  $\Delta E$  is the difference between the target function of current response and the neighboring response and  $T$  is a parameter called the temperature. At each temperature, there are repeated actions and then the temperature is gradually reduced. In the first steps, the temperature is very high in order to have the higher probability of accepting the worse responses. By gradual reduce of temperature in the final steps, the lower probability will be created for accepting the worse responses; thus, the algorithm converges to a good response. In each step, the simulated annealing algorithm considers several states in neighboring state  $s$  and makes possible decision whether to transfer the system from the state  $s$  or remain in this state. These probabilities finally lead the system to a state with lower energy. The neighbors of a state (response) are the new problem states which are created

with the changes in the current state and with regard to a pre-determined method. In summary, the simulated annealing algorithm, used in this research, can be shown according to the following figure (Figure (2)).

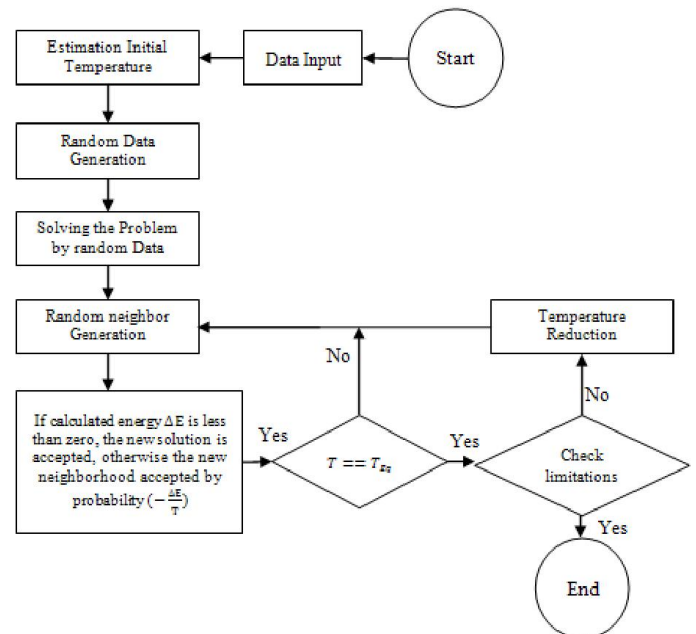


Fig 2: Applied Simulated Annealing Flowchart in this study

#### 3-1- INITIAL TEMPERATURE

Given the higher impact, which the regulation of parameters will have on the quality of final response, the parameters are set as follows. The initial temperature is one of the most important parameters which exists in the algorithm structure and plays a significant role in the acceptance or exclusion of solution in the neighborhood of Boltzmann function. Thus, the initial temperature is calculated from the following formula:

$$T_0 = -\frac{\Delta E}{\ln(p)}$$

The probability ( $p$ ) is considered about (0.01 – 0.05) and for instance, they consider the solution acceptance probability equal to 0.01.  $\Delta E$  is the difference in the amount of fitness of target function in the initial solution and neighboring solution. In order to estimate the initial temperature of algorithm in this study, all studied points are investigated and the value of target function is for all of them, and the difference of single maximal and minimal values are subtracted from each other. Based on the model estimation in this study, the initial temperature is estimated equal to  $1.7430e + .17$ .

#### 3-2-TEMPERATURE DECREASE MODEL

In this model, a linear algorithm is used in order to reduce the temperature. Therefore, according to the following equation the value of  $\alpha$  is received as a input data by the

executive at the beginning of model implementation and then during the gradual refrigeration in solving the algorithm, the value of reduced temperature will be as a multiplied value of previous temperature.

$$T_{i+1} = \alpha \times T_i$$

The value of  $\alpha$  in this study is obtained by the mathematical program executive. This value has the same response with approximately higher than 85% probability. Thus, using the values closer to 1 creates the higher time calculations, but it will have no effect on the calculations in this particular model. This is due to the higher difference among the nodes and this causes that the mathematical model in MATLAB to find the answers quickly.

### 3-3- INITIAL SOLUTION

In this study, the initial solution was randomly selected due to the limited amount points to 81 nodes and hence not taking time for optimal search.

### 3-4- NEIGHBORING SEARCH

The random method with making the random numbers was used in order to find the neighborhood in this algorithm.

### 3-5- CHOICE SCHEMA

The following measure is done in order to select the best solution from the initial solution in the population and solution in neighborhood: If the value of target function is larger in the neighborhood, it will be selected as the new solution; otherwise, the value of Boltzmann function will be calculated. Given the generated random number in the range [0,1], if the random number is smaller than the value calculated for Boltzmann, the neighboring solution is accepted, otherwise it will not be accepted.

### 3-6- CONDITION FOR FINISHING THE CALCULATIONS

In most of the optimization algorithms, reaching the desired value of calculated target function or error or comparison of two sequential values is considered as the condition for finishing the calculations. The algorithm is stopped whenever the temperature in simulated annealing algorithm is lower than the target freezing temperature. In this research, the freezing temperature near zero was considered equal to 32 in order to increase the accuracy.

### 3-7- OBTAINED RESULTS OF ANNEALING ALGORITHM

Given the input value for district 7 of great Tehran and assuming the establishment of three branches (this assumption can be changed and determined at the time of implementing the program), the output results of software include the nodes 23, 3, and 78, respectively. It should be noted that this points are preferable to others interms of location and have been obtained by the software with refrigeration coefficient (annealing algorithm parameter) higher than 0.85. The relevant points are shown schematically in the following figure (Figure (3)).

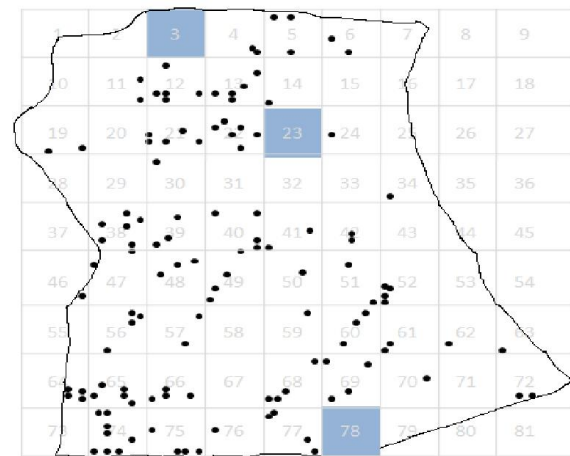


Fig .3. Sequential display of three optimal points for establishing the new branch

As shown in the Figure (3), the calculated points by the program will be resulted in logical responses because all selected points have no high numbers of rival banks and have higher population modem. This is due to the higher convergence of this issue because the number of available nodes is equal to a few 81 nodes; on the other hand, choosing each of the nodes does not affect other nodes. In other words, the nodes are separated from each other and the program should only examine all available nodes and calculate the target function and then compare it with the previous values. Furthermore, there are numerous differences among the existing nodes because the number of banks reaches 9 in some regions and equal to 0 in other regions. Moreover, these values have higher differences in the nodes in terms of other indicators such as the population and also the quality of life.. According to the Gradual Annealing Algorithm, the values of following cases were calculated: Consumers' preferences equal to ( $\gamma = 0.1 - 1.00$ ), optimal node, target function, and the ratio of service in optimal node in the refrigeration coefficient ( $SAP = 0.1 - 0.99$ ); and these values are presented in the following tables (Table (1)).

Table 1: Values of target function and coefficient of optimal node service according to different customers' preferences and constant refrigeration coefficient

SAP	0.1		
CPP	ON	OF	SR
0.1	38	550.2007	0.15
0.2	5	1.5346	0.13
0.3	5	1.5346	0.13
0.4	39	295.7874	0.06
0.5	64	54.0305	0.11
0.6	66	112.1261	0.07
0.7	22	1668.4606	0.13
0.8	46	0.5048	0.71
0.9	68	19.5541	0.05
1	68	19.5541	0.05
SAP	0.2		
0.1	20	0.5252	0.3
0.2	5	1.5346	0.13
0.3	5	1.5346	0.13
0.4	52	20.9417	0.03
0.5	38	550.2007	0.15
0.6	22	1668.4606	0.13
0.7	50	109.642	0.01
0.8	60	28.1523	0.02
0.9	57	162.1012	0.02
1	57	162.1012	0.02
SAP	0.3		
0.1	44	3.083	0.05
0.2	5	1.5346	0.13
0.3	5	1.5346	0.13
0.4	8	0.4609	0.28
0.5	22	1668.4606	0.13
0.6	77	29.3773	0.06
0.7	69	3.5019	0.03
0.8	13	3091.101	0.09
0.9	13	3091.101	0.09
1	13	3091.101	0.09
SAP	0.4		
0.1	68	19.5541	0.05
0.2	5	1.5346	0.13
0.3	5	1.5346	0.13
0.4	6	87.4554	0.01
0.5	12	3084.9887	0.09
0.6	74	74.726	0.16
0.7	14	0.4624	0.6
0.8	11	0.5195	0.86
0.9	5	1.5346	0.13
1	5	1.5346	0.13
SAP	0.5		
0.1	50	109.642	0.01
0.2	5	1.5346	0.13
0.3	5	1.5346	0.13
0.4	42	54.9561	0.02
0.5	13	3091.101	0.09
0.6	48	345.01	0.05
0.7	68	19.5541	0.05
0.8	60	28.1523	0.02
0.9	78	12.6879	0.02
1	78	12.6879	0.02
SAP	0.6		
0.1	1	0.5157	0.27
0.2	5	1.5346	0.13
0.3	5	1.5346	0.13
0.4	74	74.726	0.16
0.5	21	1460.9685	0.1
0.6	74	74.726	0.16
0.7	38	550.2007	0.15
0.8	12	3084.9887	0.09
0.9	47	202.4376	0.01
1	47	202.4376	0.01
SAP	0.7		
0.1	80	0.4639	0.47
0.2	5	1.5346	0.13
0.3	5	1.5346	0.13
0.4	56	348.065	0.06
0.5	56	348.065	0.06
0.6	65	311.6207	0.09
0.7	4	92.2209	0.05
0.8	11	0.5195	0.86
0.9	42	54.9561	0.02
1	42	54.9561	0.02
SAP	0.8		
0.1	56	348.065	0.06
0.2	31	136.6543	0.01
0.3	47	202.4376	0.01
0.4	74	74.726	0.16
0.5	56	348.065	0.06
0.6	19	12.9332	0.16
0.7	74	74.726	0.16
0.8	56	348.065	0.06
0.9	41	127.5527	0.04
1	41	127.5527	0.04
SAP	0.9		
0.1	42	54.9561	0.02
0.2	43	27.1393	0.01
0.3	16	1.5524	0.03
0.4	74	74.726	0.16
0.5	68	19.5541	0.05
0.6	61	4.9793	0.01
0.7	70	3.0516	0.01
0.8	3	209.8384	0.02
0.9	5	1.5346	0.13
1	5	1.5346	0.13
SAP	0.99		
0.1	38	550.2007	0.15
0.2	44	3.083	0.05
0.3	66	112.1261	0.07
0.4	74	74.726	0.16
0.5	49	219.3132	0.05
0.6	66	112.1261	0.07
0.7	76	19.721	0.04
0.8	70	3.0516	0.01
0.9	76	19.721	0.04
1	76	19.721	0.04



SAP	0.1		
CPP	ON	OF	SR
0.1	38	550.2007	0.15
0.2	20	0.5252	0.3
0.3	44	3.083	0.05
0.4	68	19.5541	0.05
0.5	50	109.642	0.01
0.6	1	0.5157	0.27
0.7	80	0.4639	0.47
0.8	56	348.065	0.06
0.9	42	54.9561	0.02
0.99	38	550.2007	0.15
SAP	0.2		
0.1	5	1.5346	0.13
0.2	5	1.5346	0.13
0.3	5	1.5346	0.13
0.4	5	1.5346	0.13
0.5	5	1.5346	0.13
0.6	5	1.5346	0.13
0.7	5	1.5346	0.13
0.8	31	136.6543	0.01
0.9	43	27.1393	0.01
0.99	44	3.083	0.05
SAP	0.3		
0.1	5	1.5346	0.13
0.2	5	1.5346	0.13
0.3	5	1.5346	0.13
0.4	5	1.5346	0.13
0.5	5	1.5346	0.13
0.6	5	1.5346	0.13
0.7	5	1.5346	0.13
0.8	47	202.4376	0.01
0.9	16	1.5524	0.03
0.99	66	112.1261	0.07
SAP	0.4		
0.1	39	295.7874	0.06
0.2	52	20.9417	0.03
0.3	8	0.4609	0.28
0.4	6	87.4554	0.01
0.5	42	54.9561	0.02
0.6	74	74.726	0.16
0.7	56	348.065	0.06
0.8	74	74.726	0.16
0.9	74	74.726	0.16
0.99	74	74.726	0.16
SAP	0.5		
0.1	64	54.0305	0.11
0.2	38	550.2007	0.15
0.3	22	1668.4606	0.13
0.4	12	3084.9887	0.09
0.5	13	3091.101	0.09
0.6	21	1460.9685	0.1
0.7	56	348.065	0.06
0.8	56	348.065	0.06
0.9	68	19.5541	0.05
0.99	49	219.3132	0.05

SAP	0.6		
CPP	ON	OF	SR
0.1	66	112.1261	0.07
0.2	22	1668.4606	0.13
0.3	77	29.3773	0.06
0.4	74	74.726	0.16
0.5	48	345.01	0.05
0.6	74	74.726	0.16
0.7	65	311.6207	0.09
0.8	19	12.9332	0.16
0.9	61	4.9793	0.01
0.99	66	112.1261	0.07
SAP	0.7		
0.1	22	1668.4606	0.13
0.2	50	109.642	0.01
0.3	69	3.5019	0.03
0.4	14	0.4624	0.6
0.5	68	19.5541	0.05
0.6	38	550.2007	0.15
0.7	4	92.2209	0.05
0.8	74	74.726	0.16
0.9	70	3.0516	0.01
0.99	76	19.721	0.04
SAP	0.8		
0.1	46	0.5048	0.71
0.2	60	28.1523	0.02
0.3	13	3091.101	0.09
0.4	11	0.5195	0.86
0.5	60	28.1523	0.02
0.6	12	3084.9887	0.09
0.7	11	0.5195	0.86
0.8	56	348.065	0.06
0.9	3	209.8384	0.02
0.99	70	3.0516	0.01
SAP	0.9		
0.1	68	19.5541	0.05
0.2	57	162.1012	0.02
0.3	13	3091.101	0.09
0.4	5	1.5346	0.13
0.5	78	12.6879	0.02
0.6	47	202.4376	0.01
0.7	42	54.9561	0.02
0.8	41	127.5527	0.04
0.9	5	1.5346	0.13
0.99	76	19.721	0.04
SAP	1		
0.1	68	19.5541	0.05
0.2	57	162.1012	0.02
0.3	13	3091.101	0.09
0.4	5	1.5346	0.13
0.5	78	12.6879	0.02
0.6	47	202.4376	0.01
0.7	42	54.9561	0.02
0.8	41	127.5527	0.04
0.9	5	1.5346	0.13
0.99	76	19.721	0.04

## 4 -SUGGESTIONS FOR FUTURE RESEARCH

Given the fact that the response of issue determines the range of optimal points not the optimal point, a sub-model can be developed in the future research in addition to model developed in this study, so the optimal points will be determined after specifying the optimal location scope. For instance, the mathematical model, which examines several special features in desired scope, can be applied in order to determine the optimal points in optimal range. Also, the maximized profit of established branches is the target function on this study. However, if the multi-purpose mathematical models are used, the target function can be extended in a way that it meets the criteria of choosing the customers such as the minimum travelling time, and minimum waiting time as well as maximizing the bank profit. Therefore, it is suggested applying the mathematical model with multi-purpose function in future studies.

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